LAB 9.1
Taxicab Versus Euclidean Distance

Name(s) ______________________

- **Equipment:** Geoboard, graph or dot paper

If you can travel only horizontally or vertically (like a taxicab in a city where all streets run North-South and East-West), the distance you have to travel to get from the origin to the point (2, 3) is 5. This is called the taxicab distance between (0, 0) and (2, 3). If, on the other hand, you can go from the origin to (2, 3) in a straight line, the distance you travel is called the Euclidean distance, or just the distance.

Finding taxicab distance: Taxicab distance can be measured between any two points, whether on a street or not. For example, the taxicab distance from (1.2, 3.4) to (9.9, 9.9) is the sum of 8.7 (the horizontal component) and 6.5 (the vertical component), for a total of 15.2.

1. What is the taxicab distance from (2, 3) to the following points?

a. (7, 9)

b. (–3, 8)

c. (2, –1)

d. (6, 5.4)

e. (–1.24, 3)

f. (–1.24, 5.4)

3. Find as many geoboard pegs as possible that are at a distance 5 from (5, 5). Record your findings on graph or dot paper.

(Using taxicab distance!)

Discussion

A. Find a formula for the taxicab distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Call the distance $T(P_1, P_2)$. (**Hint:** Start by figuring out a formula for the case where the points are on a common horizontal or vertical line. The formula should work whether P_1 or P_2 is named first.)

B. In Euclidean geometry, for three points A, B, and C, we always have $AB + BC \geq AC$. This is called the *triangle inequality*. Does this work in taxicab geometry? In other words, do we have $T(A, B) + T(B, C) \geq T(A, C)$? If so, in what cases do we have equality?

C. Which is usually greater, taxicab or Euclidean distance? Can they be equal? If so, in what cases?

D. Explain why the answers to Problem 3a are located on what may be called a *taxi-circle*.

E. How would you define taxicab distance from a point to a line? Consider different cases.

LAB 9.6
Taxicab Geometry

Name(s) ______________________

■ **Equipment:** Dot or graph paper

Problems 2–5 involve taxicab distances to two points.

2. Sketch the set of points that are equidistant from two points A and B in the following cases.
   a. $A(0, 0)$ and $B(6, 0)$
   b. $A(0, 0)$ and $B(4, 2)$
   c. $A(0, 0)$ and $B(3, 3)$
   d. $A(0, 0)$ and $B(1, 5)$
3. Find the set of points P such that $PA + PB = 6$ in the cases listed in Problem 2.
4. Find the set of points P such that $PA + PB = 10$ in the cases listed in Problem 2.
5. Find the set of points P such that $PA/PB = 2$ in the cases listed in Problem 2. (Careful! This last case is particularly tricky.)

Problems 6–9 are about taxi-circles.

6. The number π is the ratio of the perimeter of a circle to its diameter. In Euclidean geometry, $\pi = 3.14159 \ldots$. Find the value of taxi-π.

7. Given the line l with equation $y = 3x$ and the point $P(3, 5)$, construct a taxi-circle with radius 3 that passes through P and touches l in exactly one point. (There are two such circles.)

8. Explain how to find the center of a taxi-circle that goes through two points A and B and, once you have the center, how to sketch the circle. Give examples based on the cases listed in Problem 2.

9. In Euclidean geometry, three noncollinear points determine a unique circle, while three collinear points determine no circle. In taxicab geometry, the situation is somewhat more complicated. Explore different cases, and try to find out when three points determine no circle, one circle, or more than one circle.

Discussion

A. Given two points A and B, how would you find a third vertex P such that ΔABP is taxi-isosceles? (**Hint:** There are two cases: $PA = PB$ and $PA = AB$. And, of course, A and B can be positioned in various ways, such as the ones listed in Problem 2.)

B. Given two points A and B, find the set of points P such that $PA + PB$ is minimal. Investigate the same question for three or even more points.