The Power of Wishful Thinking
And other problem solving strategies

Hints for problems

Warm up Problem: The numbers 1 through 100 are written on the board. Each minute, you choose any two numbers x and y Erase x and y and write $xy + x + y$ instead. After 99 minutes, there will be only one number left. Find all possible numbers that can occur at the end of this process.

What would we wish for here?

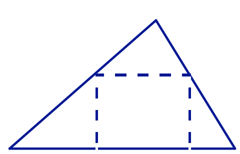
- We might wish there were fewer numbers to begin with. What if there were only 2 numbers? Only 3? Only 4? What do you notice in each case? Do you see a pattern appearing?
- We might wish that we didn't have to worry about the order in which we choose the numbers. Our investigation with smaller numbers seems to indicate that the order doesn't matter. How can we check this?
- We might wish that we could factor $xy + x + y$. How close can we get to factoring this? What are we missing? Can we fix the problem? Notice the symmetry of the expression. How can we use that? What happens if we consider the next step, when we represent three numbers with variables x, y, z?

Some things to think about on the remaining problems:

1. Find the sum of the digits in the product 55,555,555,555 X 999,999,999,99
    - We might wish that each of the factors had fewer digits. Can we find easier multiplication problems that will give us insight into what this product will look like?
    - We might wish that one or more of the factors were something easier to multiply. Can we change one or both of them to achieve this? If so, can we change back to the original problem?

2. Show that 100^{100} can be written as the sum of 4 perfect cubes.
    - We might wish that the exponent were smaller.
    - We might wonder what numbers can be written as the sum of 4 perfect cubes.

3. Show that a square can be inscribed in any triangle, *i.e.* show that there is a square with two vertices one side of the triangle and the other two vertices on the remaining two sides of the triangle.



- We might wish that the square only had to have three vertices on the triangle instead of four.
- We might wish that the inscribed quadrilateral only needed to be a rectangle rather than a square.
- We might wonder how much (or little) information we need to know about the square in order to be able to construct it.
- We might wish that we had to draw a triangle around the square instead of putting the square in the triangle.
- We might wish that we could use algebra instead of geometry. How can we change a geometric problem into an algebraic one?