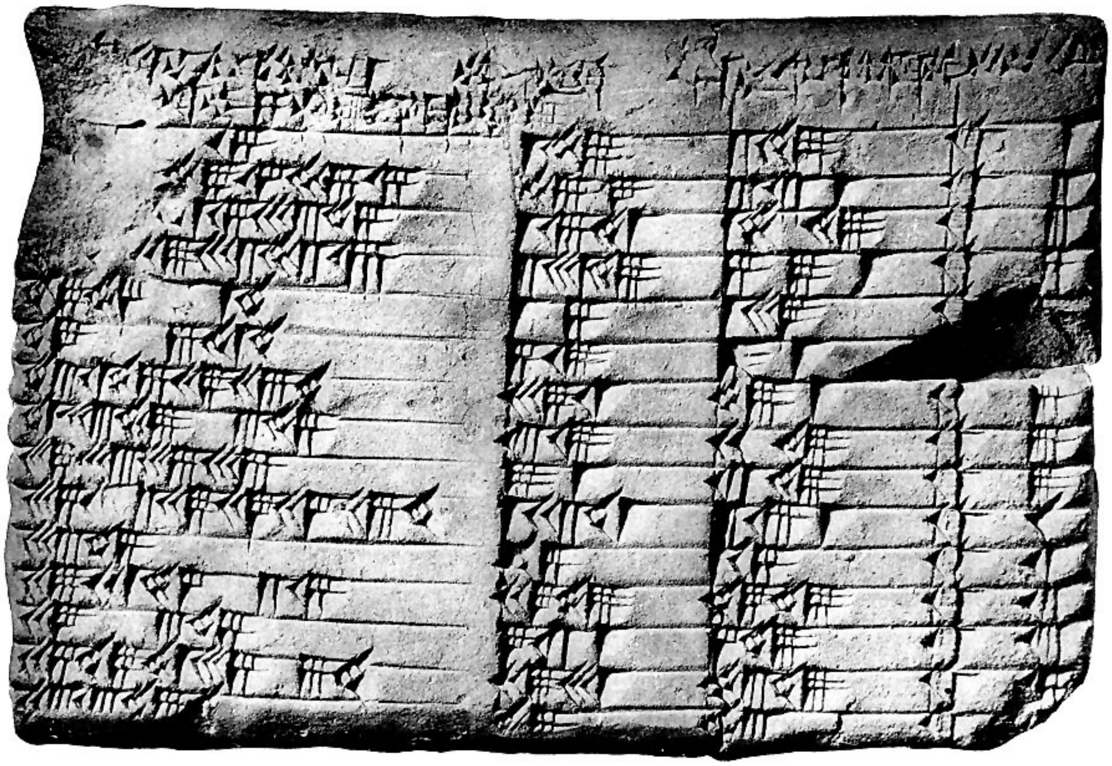
Plimpton 322

http://www.math.ubc.ca/~cass/courses/m446-03/pl322/322.jpg, color image:
http://www.maa.org/press/periodicals/convergence/mathematical-treasures-plimpton-322



(1:)59:00:15	1:59	2:49	1
(1:)56:56:58:14:50:06:15	56:07	1:20:25	2
(1:)55:07:41:15:33:45	1:16:41	1:50:49	3
(1:)53:40:29:32:52:16	3:31:49	5:09:01	4
(1:)48:54:01:40	1:05	1:37	5
(1:)47:06:41:40	5:19	8:01	6
(1:)43:11:56:28:26:40	38:11	59:01	7
(1:)41:33:45:14:03:45	13:19	20:49	8
(1:)38:33:36:36	8:01	12:49	9
(1:)35:10:02:28:27:24:26:40	1:22:41	2:16:01	10
(1:)33:45	45	1:15	11
(1:)29:21:54:02:15	27:59	48:49	12
(1:)27:00:03:45	2:41	4:49	13
(1:)25:48:51:35:06:40	29:31	53:49	14
(1:)23:13:46:40	56	1:46	15

Using the Babylonian sexagesimal system, the entries in the 2nd and 3rd columns of row 1 can be interpreted as $(1 \cdot 60 + 59)$ and $(2 \cdot 60 + 49)$. Fill in the corresponding decimal values for the second and third columns. Do you notice any patterns?

Right Triangles, the Unit Circle, and Pythagorean Triples

Some people have seen Plimpton 322 as a sort of trig table. List several possibilities for integer-sided right triangles. That is, list Pythagorean triples (a,b,c) that satisfy

$$a^2 + b^2 = c^2$$

How many Pythagorean triples can you produce for which the three numbers share no common factor? Is it possible to have any Pythagorean triples that are all even numbers? All odd numbers? Exactly one odd and two even?

Plimpton 322 (ca 1800 BCE) lists what correspond to the short leg and hypotenuse of 15 right triangles with integer sides, in order of small angle, decreasing in approximately 1° steps. How can one find so many Pythgorean triples?

For each Pythagorean triple (a, b, c), consider the related triple $\left(\frac{a}{c}, \frac{b}{c}, 1\right)$.

- Does this remind you of any math you've already studied?
- Does every Pythagorean triple correspond to a point on the graph of $x^2 + y^2 = 1$?
- Suppose that the point (x, y) lies on graph of $x^2 + y^2 = 1$ and that both x and y are rational. How can you produce a Pythagorean triple from x and y?
- What is the equation of the line passing through $(0,-1)$ with slope 2? Where does this line intersect the unit circle $x^2 + y^2 = 1$, besides at the point $(0,-1)$? Does this give you a familiar Pythagorean triple?
- What about the line through $(0,-1)$ and with slope 3 and its intersection with the unit circle?

Pythagorean triples are related to the "rational points" on the unit circle.

- How does each first-quadrant rational point on the unit circle give us a Pythagorean triple?
- If m is a rational number, explain why the equation

  $$x^2 + (mx - 1)^2 = 1$$

  is a quadratic equation with rational coefficients.
- Justify: If m is a nonzero rational number, the line $y = mx - 1$ intersects the unit circle at two rational points.
- Explain how you can find any number of distinct Pythagorean triples.

Extra challenge: Assume that $m = \frac{p}{q}$, where p and q are positive integers and $p > q$. Using your strategy for producing Pythagorean triples, find a Pythagorean triple in terms of p and q. (Neugebaurer believed the that the Babylonians essentially used such expressions.)

Reciprocal Pairs

Eleanor Robson argued in 2001 that Plimpton 322 actually had to do with computing reciprocal pairs x and $\frac{1}{x}$ based on their difference: $x - \frac{1}{x} = c$ for fifteen different values of c. (Presumably those numbers would have been shown in a left-side column that has since broken off.)

Nowadays, to solve the equation for x, we'd probably multiply through by x and solve the resulting quadratic equation. If you solve the quadratic by the method of completing the square (which can be done geometrically), you may be able to decode the following steps that Robson says the Babylonians used.

Step 1: compute $v_1 = \frac{c}{2}$. (This value appears in column 2.)
Step 2: compute $v_2 = 1 + v_1^2$. (This value appears in column 1.)
Step 3: computer $v_3 = \sqrt{v_2}$. (This values appears in column 3.)
Then $x = v_3 + v_1$ and $\frac{1}{x} = v_3 - v_1$. (Double-check this.)

“In short, Robson suggests that the tablet would probably have been used by a teacher as a problem set to assign to students.” https://en.wikipedia.org/wiki/Plimpton_322
http://www.maa.org/sites/default/files/pdf/upload_library/22/Ford/Robson105-120.pdf