

GeoGebra Explorations and the Parallel Postulate

1. Given a segment \overline{AB} , and a point D , construct a square with one vertex at D and side length AB . Write out the steps of your construction and prove that the resulting figure is a square. Where, if ever, did you use the Parallel Postulate in your proof? Can you prove that the figure is a square without using the Parallel Postulate? How much can you prove about the figure using only results in neutral geometry? (In neutral geometry, the Parallel postulate not assumed to be true nor is assumed to be false.)
2. Given a line l and a point P not on l , construct a line through P parallel to l . Justify that your construction works. Where, if ever, did you use the Parallel Postulate?

In the next few problems, you will use the following definition of a *parallelogram*: A parallelogram is a quadrilateral in which each pair of opposite sides is parallel.

3. Given a segment \overline{AB} and a point D , construct a quadrilateral $DEFG$ such that $\overline{DE} \parallel \overline{FG}$ and $DE = FG = AB$. Be sure that your quadrilateral is as general as possible while still satisfying the given conditions. What can you say about the quadrilateral? Based on your example, complete and prove the following statement:
A quadrilateral is a parallelogram if and only if one pair of opposite sides is _____.
Does your proof use the Parallel Postulate? If so, where?
4. Using the quadrilateral you constructed in part 3, draw in the two diagonals. What do you observe? Based on your observations, complete and prove the following statement:
A quadrilateral is a parallelogram if and only if the diagonals _____.
Does your proof use the Parallel Postulate? If so, where?
5. Vary the quadrilateral from part 4 and observe what happens when the quadrilateral is a rectangle. Based on your observations, complete and prove the following statement:
A quadrilateral is a rectangle if and only if the diagonals _____.
Does your proof use the Parallel Postulate? If so, where?
6. Complete and prove the following statements:
(a) A quadrilateral is a rhombus if and only if the diagonals _____.
(b) A quadrilateral is a square if and only if the diagonals _____.
Do your proofs use the Parallel Postulate? If so, where?