

Knights and Knaves:
A journey to the land of logic

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Tweedledee: I know what you're thinking, but it isn't so, nohow.

Tweedledum: Contrariwise, if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic.

-Lewis Carroll, Through the Looking Glass

Logic provides the framework that allows us to agree on what is, or is not, a valid argument. Without getting into a detailed study of the rules of logic, it is possible to deepen our students' understanding of these rules by offering them some well-chosen puzzles that highlight important ideas. The following puzzles all come from a lovely book entitled What is the Name of this Book: The riddle of Dracula and other logical puzzles, written by Raymond Smullyan, a mathematician, philosopher, magician, and author.

Part 1. Knights and Knaves: You have just arrived on an island inhabited by two kinds of people: knights who always tell the truth, and knaves who always lie. Every inhabitant of the island is either a knight or a knave, and everyone knows which inhabitants are knights and which are knaves. You are a stranger on the island, and you do not know who is a knight and who is a knave.

Scenario 1: You come across two inhabitants of this island, A and B. A makes the following statement: "At least one of us is a knave." What, if anything, can you determine about A and/or B? Be prepared to defend your answer!

Scenario 2: Same situation as above, except this time A states, "Either I am a knave or B is a knight." What, if anything, can you determine about A and/or B?

Scenario 3: Suppose that instead, A states: “I am a knave, but B isn’t.” Now, what, if anything, can be deduced?

Scenario 4: We have two people, A and B, each of whom is either a knight or a knave. A makes the following statement: “If I am a knight, then so is B.” Can it be determined what A and B are?

Scenario 5: Same situation as above, but this time A makes the following statement: “If B is a knight, then I am a knave.” Can it be determined what A and B are?

Scenario 6: It is rumored that there is gold on the island of knights and knaves. You ask an inhabitant if there really is gold on the island. He responds, “There is gold on this island if and only if I am a knight.”

- (a) Can it be determined whether the speaker is a knight or a knave?
- (b) Can it be determined whether or not there is gold on the island?

Scenario 7: This time, suppose that you come across three people, A, B, and C. You ask A, “How many knights are there among the three of you?” A responds, but he mumbles so badly that you cannot understand him. So you turn to B and ask, “What did A say?” B responds, “A said that there is one knight among us.” Then C says: “Don’t believe B; he is lying!”

Can you tell what B is? What about C? What about A?

Scenario 8: Again we have three people, A, B, and C, each of whom is either a knight or a knave. A and B make the following statements:

- A: All of us are knaves.
- B: Exactly one of us is a knight.

What, if anything, can be determined about A, B, and C?

Scenario 9: Suppose instead, A and B said the following:

- A: All of us are knaves.
- B: Exactly one of us is a knave.

Now, what, if anything, can be determined about A, B, and C?

Scenario 10: We again have three people, A, B, and C, each of whom is either a knight or a knave. Two people are said to be of the *same type* if they are both knights or both knaves. A and B make the following statements:

A: B is a knave.

B: A and C are of the same type.

What is C?

Scenario 11: Again three people, A, B, and C. A states, "B and C are of the same type." You then ask C, "Are A and B of the same type?" Is it possible to deduce what C will answer?

Scenario 12: Two individuals, X and Y, are being tried for participation in a robbery. A and B are court witnesses, and each of A and B is either a knight or a knave. The witnesses make the following statements:

A: If X is guilty, then so is Y.

B: Either X is innocent or Y is guilty.

Are A and B necessarily of the same type?

Part 2. Knights, Knaves, and Normals: On a neighboring island there are three types of people: knights who always tell the truth, knaves who always lie, and normals who sometimes lie and sometimes tell the truth.

Scenario 1: You have just left the island of knights and knaves and land on this neighboring island. You meet three people, A, B, and C, one of whom is a knight, one a knave, and one a normal, although you do not know which is which. They make the following statements:

A: I am normal.

B: What A said is true.

C: I am not normal.

Is it possible to deduce what A, B, and C are?

Scenario 2: Two people A and B, each of whom is either a knight, or knave, or normal, make the following statements:

A: B is a knight.

B: A is not a knight.

Prove that at least one of them is telling the truth, but it is not a knight.

Scenario 3: On this island of knights, knave, and normal, knaves are said to be of *the lowest rank*, normals are of *middle rank*, and knights of *highest rank*. Two people, A and B, each of whom is a knight, knave, or normal, make the following statements:

A: I am of lower rank than B.

B: That's not true!

Can the ranks of A or B be determined? Can it be determined whether the statement made by A is true or false? What about the statement made by B?

Scenario 4: Given three people, A, B, and C, one of whom is a knight, one a knave, and one normal. A and B make the following statements:

A: B is of higher rank than C.

B: C is of higher rank than A.

Then C is asked, "Who has higher rank, A or B?" What does C answer?

Part 3. Yes, you are innocent, but can you prove it? In the puzzles that follow, you are an inhabitant of the island of knights, knaves, and normals. A crime has been committed on the island, and for some strange reason it is suspected that you are the criminal. You are brought to court and tried. You are allowed to make only one statement in your own behalf. Your purpose is to convince the jury that you are innocent.

Scenario 1: Suppose that it is known that the criminal is a knave. Suppose also that you are a knave (although the court does not know this) but you are nevertheless innocent of this crime. You are allowed to make only one statement. Your purpose is to convince the jury that you are innocent of the crime. What would you say?

Scenario 2: Suppose that it is known that the criminal is not normal – he is either a knight or a knave. You yourself are innocent. What statement could you make to which could be made by either a knight, a knave, or a normal in your position, which would convince the jury that you are innocent?

Scenario 3: Again it is known that the criminal is not normal. Suppose that 1) you are innocent and 2) you are not a knave. Is there a single statement you could make to convince the jury of both of these facts?

Part 4. How I became rich: I found out about three neighboring islands, A, B, and C. I knew that there was gold buried on at least one of the three islands, but I didn't know which ones. Islands B and C were uninhabited; island A was inhabited by knights and knaves, and there was a possibility that there were some normals on the island, but I didn't know whether there were any normals or not. I had the good fortune to find the map of the islands left by a famous, but capricious pirate, Captain Marston – the pirate who buried the gold. The message he left on the map consisted of two sentences:

1. There is no gold on Island A.
2. If there are any normal on Island A, then there is gold on two of the islands.

Well, I rushed over to Island A; I knew that the natives there knew all about the gold situation. The King of the island guessed what I was up to and told me that I was allowed to ask only one question of one native that I chose at random. I would have no way of knowing if the native was a knight, knave, or normal. What question could I ask such that, upon hearing the answer, I could point to one of the islands and be sure that there was gold on that island?

Part 5. The Island of Zombies: On a certain island, half the inhabitants are human and half are zombies. The zombies of this island, look and sound just like humans, but the difference is that the humans always tell the truth and the zombies always lie. Both humans and zombies on the island understand English, but the custom on the island absolutely forbids them to speak anything other than their native language. Therefore, whenever you ask them a yes/no question, they reply “Bal” or “Da” – one of which means *yes* and the other *no*. The trouble is that we do not know which of “Bal” or “Da” means *yes* and which means *no*.

Scenario 1: You meet a native of this island and ask him, “Does ‘Bal’ mean *yes*?” He replies, “Bal.”

- (a) Is it possible to infer what “Bal” means?
- (b) Is it possible to infer whether he is a human or a zombie?

Scenario 2: You hear a rumor that there is gold on the island. You arrive on the island, and before you start excavating, you want to know whether the rumor is true or not. The natives all know whether or not there is gold on the island. How, in one question to any one of the natives, can you find out?

Part 6. From the files of Inspector Craig: Inspector Leslie Craig of Scotland Yard has kindly consented to release some of his case histories for those interested in the application of logic to the solution of crimes.

Scenario 1: An enormous amount of loot has been stolen from a store. The criminal (or criminals) took the loot away in a car. Three well-known criminals, A, B, and C, were brought to Scotland Yard for questioning. The following facts were ascertained:

1. No one other than A, B, and C was involved in the robbery.
2. C never pulls a job without using A (and possibly others) as an accomplice.
3. B does not know how to drive

Is A innocent or guilty?

Scenario 2: “What do you make of these three facts?” asked Inspector Craig to Sergeant McPherson.

1. If A is guilty and B is innocent, then C is guilty.
2. C never works alone.
3. A never works with C.
4. No one other than A, B, or C was involved, and at least one of them is guilty.

The sergeant scratched his head and said, “Not much, I’m afraid, sir. Can you infer from these facts which ones are innocent and which ones are guilty?”

“No,” responded Craig, “but there is enough material here to definitely indict one of them.”

Which one is necessarily guilty?

Scenario 3: This case took place on the island of knights, knaves, and normals. Three inhabitants of the island A, B, and C are on trial for a crime. It is known that the crime was committed by only one of them. It is also known that the one who committed the crime is a knight, and the only knight among them. The three defendants made the following statements:

- A: I am innocent.
- B: That is true.
- C: B is not normal.

Which one is guilty?

Scenario 4: This, the most interesting case of all, bears a superficial resemblance to the above, but is really quite different. It also took place on the island of knights, knaves, and normals.

The principal actors in this case were the defendant, the prosecutor, and the defense attorney. The first baffling thing was that it was known that one of them was a knight, one a knave, and one normal, though it was not known which was which. Even stranger, the court knew that if the defendant was not guilty, then the guilty one was either the defense attorney or the prosecutor. It was also known that the guilty one was not a knave. The three made the following statements in court:

Defendant: I am innocent.

Defense Attorney: My client is indeed innocent.

Prosecutor: Not true, the defendant is guilty.

These statements certainly seemed natural enough. The jury convened, but could not come to any decision; the above evidence was insufficient. Now this island was a British possession at the time, hence the government wired to Scotland Yard asking whether they could send Inspector Craig to come over to help settle the case.

Several weeks later Inspector Craig arrived, and the trial was reconvened. Craig said to himself, "I want to get to the bottom of this! He wanted to know not only who was guilty, but also which one was the knight, which the knave, and which the normal. So he decided to ask just enough questions to settle these facts. First, he asked the prosecutor, "Are you, by any chance, the guilty one?" The prosecutor answered. Inspector Craig thought a while and then he asked the defendant, "Is the prosecutor guilty?" The defendant answered, and Inspector Craig knew everything.

Who was guilty, who was normal, who was the knight, and who was the knave?