Any Conic Section (So Long as It is a Parabola) AIM MTC

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Imagine a very thirsty and hungry fly flying above a plane. It notices a river (an absolutely straight line) and a pile of food (of course, it's a point off the line which is the river) and tries to land. Unfortunately, it's a totally logical creature and it's equally attracted to water and food. After an anguishing moment of indecision it lands at a spot which is at the exact same distance from both attractors. But alas, the fly cannot reach either of them! So instead of sitting still it attempts to move. Sure thing, it remains equidistant from both the river and the food pile at every spot of its path.

1. (a) Draw the poor fly's path. Use the grid paper and devise a method which produces points at exactly the same distances from one chosen (horizontal) line and a chosen (lattice) point above this line.
   (b) Explain your method to your neighbor(s).

2. Given a point on the fly's path, how do you find the tangent line at that point?

The fly's path described above – the set of all points in the plane which are equidistant from a given line in the plane and a fixed point off this line – is called a *parabola*. The fixed point is called a *focus* and the line is called the *directrix*. An ellipse and hyperbola have a number of similar descriptions - either using foci and directrices, or only foci. But everybody knows that their collective name is conic sections, and rightly so since they can be obtained by intersecting a doubly infinite cone with variously tilted planes. It's an interesting question to ask – how do we know that the curves defined by means of plane distances and the curves obtained by slicing a 3-d cone are indeed the same creatures? The easiest way to answer this question (well, at least the easiest that we know of) is by using Dandelin spheres. (Do it when you have time – it's delightful!)

Now get back to a parabola.

3. What happens with a light ray which enters a parabola parallel to its axis and then bounces off the parabola?

4. Lets intersect a parabola with a straight line. We obtain a flat region (called parabolic segment). What is its area?

This last problem was first solved by Archimedes about 19 hundred years before calculus (and about 21 hundred years before fractal geometry).

5. For the constructions below, use a compass and an unmarked straight edge. Construct the center of (a) a circle; (b) an ellipse; (c) a hyperbola; (d) a parabola.

6. (From *Geometry of Conics*, by A. V. Akopyan and A. A. Zaslavsky, AMS, Mathematical World, vol.26)

Two travelers move along two straight roads with constant speed. Prove that the line connecting them is always tangent to some parabola. (The roads are not parallel and travelers pass the intersection at different times.)