What is easier – (simple) counting, geometry, or algebra?

1. *[Gabriella Pinter]* Sixteen unit squares are arranged in a square array. What is the maximum number of diagonals that can be drawn in these unit squares so that no two diagonals share a common point (including endpoints)?

2. *[Putnam, 2016]* Suppose that S is a finite set of points in the plane such that the area of triangle ΔABC is at most 1 whenever A, B, and C are in S. Show that there exists a triangle of area 4 that (together with its interior) covers the set S.

3. *[Based on Putnam, 2016]* Define a positive integer n to be *squarish* if either n is a perfect square or the distance from n to the nearest perfect square is a perfect square. For example, 2016 is squarish, because the nearest perfect square to 2016 is $45^2 = 2025$ and $2025 - 2016 = 9$ is a perfect square. (Of the positive integers between 1 and 10, only 6 and 7 are not squarish.)
   For a positive integer N, let S(N) be the number of squarish integers between 1 and N, inclusive. Find $S(n^4)$ for any positive integer n.