Grid Paper Exploration

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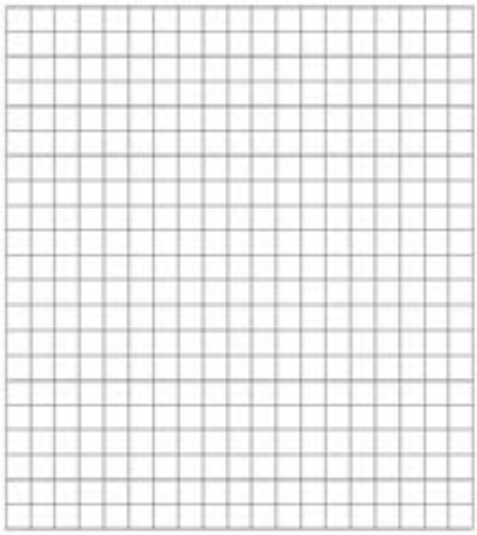
CONCEPTS: Geometry, Algebraic Thinking, Functions
SKILLS: Estimating the number of squares on a grid; writing and solving a simpler problem; collecting, organizing, and analyzing data; using the Pythagorean Theorem
MATH CONTENT STANDARDS: 6.G.1, 6.EE.2, 6.EE.6; 7.G.6, 7.EE.4; 8.G.8, 8.F.1
MATH PRACTICE STANDARDS: 1, 3, 8
GRADES: 6–8
MATERIALS: Grid paper, pencils, straightedges; whiteboards, index cards, poster paper



BACKGROUND

"When I grew up in the Soviet Union, all we used for math was grid paper. Grid paper leads to discovery." This is how Tatiana Shubin, San Jose State University, begins her lesson demonstrating the myriad wonderful mathematics questions that arise from a simple sheet of grid paper. This lesson fits beautifully with the Standards for Mathematical Practice, since it offers opportunities to practice perseverance, make viable arguments, and look for repeated reasoning, while providing numerous opportunities for extensions. The only necessary materials are grid paper, pencils and, possibly, a straightedge.

PROCEDURE

Introducing the Activity

Have students work in groups, with each student having his or her own sheet of grid paper. Instruct students to stare at their own grid paper for seven minutes and to think about what questions come to mind as they stare at the grid paper. Tell students that they should quietly write down their own questions—as many as they can—until you call time at the end of seven minutes. Move from table to table to make sure students are engaged and recording questions. Give positive reinforcement and prompt them to write down any additional questions they may have.

At the end of seven minutes, have each student share his or her own questions with the other students in the table group. Have each group then select one or two questions that they find interesting and record the questions on whiteboard, index cards, or poster paper. At this point, students should begin to see that there are many questions hidden on grid paper!

Exploring the Questions

Explore with the class the variety of questions the groups produced, such as:

1. How long can you stare at the paper without blinking?
2. How many vertical and horizontal lines are there?
3. How many squares are there?
4. How many rectangles are there?
5. How many 2 x 2 squares could be shaded without touching one another?
6. How many n x n squares fit on the paper?
7. Choose two points, A and B, with B below and to the right of A. How many ways can we travel on the grid lines (moving only down or right) from A to B?

Through these questions, students will generate their own lines of mathematical inquiry and will decide for themselves which questions are interesting. No one has made a mistake and many students are engaged.

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Talk about the different kinds of problems they have created and lead them where you want them to go! One question will almost certainly be about how many squares are on a sheet of grid paper. Let us choose to solve that problem.

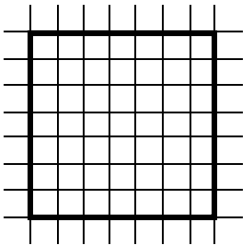
Solving the Grid Problem

Estimation as a Problem-Solving Tool

Ask students to estimate the number of squares they think are on the grid paper and write it down on a piece of paper. Then have them write a number that they are sure is too low for the number of squares and a number they are convinced is too high for the number of squares. Remind students that the grid contains different sized squares since edge lengths can be any number of grid units. Record the student estimates and then discuss the range of answers and introduce the concept of solving a simpler problem.

A Simpler Problem

Explain that, instead of figuring out how many squares are on the whole page, we will start with a smaller grid, for example, a 7 x 7 grid. Have each student clearly draw a border around a 7 x 7 grid on the grid paper. Remind them to record their thinking and organize their data so others can understand what they are doing. Then have students work individually and record their work and solutions in their own notebooks. Wander around the room and assist students where needed. If they do not see a pattern, ask how they might make the problem even simpler, such as a 2 x 2 or 3 x 3 grid, to see if a pattern emerges. Try not to be too helpful since you want students to think and reason it out on their own.



After 5–10 minutes working alone, have students work collaboratively at their tables. Explain that they need to construct viable arguments and critique the reasoning of others while their group comes to a consensus about the solution to the problem.

Sharing Solutions

After several minutes of small group work, have the class discuss the solutions from the small groups. Did groups agree or disagree?

Have students from some of the groups explain their solutions (either in part or in whole). Emphasize the importance of organizing the data, possibly by recording each group's answers in a table:

Size of Square	Number of Squares on Grid
1 x 1	49
2 x 2	36
3 x 3	25
4 x 4	16
5 x 5	9
6 x 6	4
7 x 7	1
TOTAL	140

Total is 140 squares . . . or is it? As you will see in the extensions, there may be more squares we have not yet identified!

INDEPENDENT PRACTICE

If you want your students to practice lessons from class, you may choose to assign some of the following questions for homework:

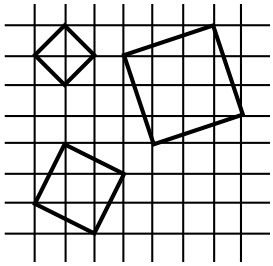
1. How many total squares in a 4 x 4 grid? 8 x 8 grid? 12 x 12 grid?
2. How many total squares in a 4 x 6 grid? 5 x 10 grid? 6 x 12 grid?
3. How many total squares were on your grid paper?
4. How many total squares in an m x m grid? An m x n grid?

EXTENSIONS

- By having students create with their own questions at the start of this lesson, we set the stage for many potential extensions. Any of the other questions that students generated could be explored as extensions.
- During this lesson we counted squares using the grid lines on our paper to form the sides of the squares. Suppose we alter our perspective slightly—instead of using grid lines to form the edges, let us use grid points and define "grid points" as the in-

tersection of two grid lines. Now count the number of squares inside a 7 x 7 square using grid points as the vertices for each square.

- In addition to all the squares students found using grid lines, we can also make "tilted" squares, for example, a square with a side length of $\sqrt{2}$ or $\sqrt{5}$. A rich discussion can ensue about squares with irrational side lengths and whole-number areas. Students will use the Pythagorean theorem to find side lengths and even generate a proof of the Pythagorean theorem, as the tilted square is enlarged to inscribe within the 7 x 7 grid.



Author's note: This article was written with substantial assistance from Hana Silverstein and Tatiana Shubin during a summer immersion workshop conducted by the American Institute of Mathematics. Their math circle for teachers has been a source of rejuvenation and wonderful mathematical frustrations.