Some Problems for Fun
Math Teachers' Circle
Alan Schoenfeld
March 25, 2017

1. What is the sum of the coefficients of $(x + 1)^{43}$?

2. What is the sum of the coefficients of

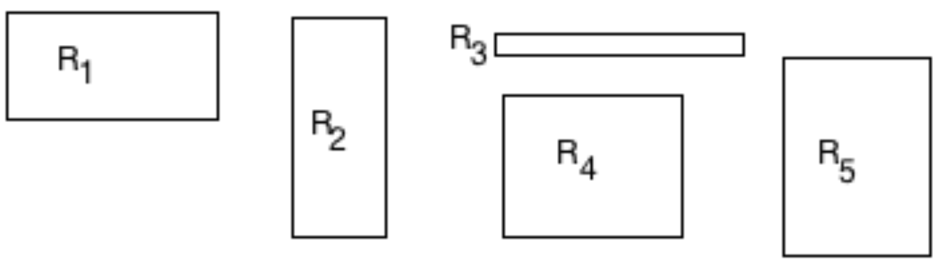
$$(7x^5 + 22x^4 + 19x^3 - 4x^2 + 9x - 4)(12x^5 – 7x^4 + 4x^3 - 2x^2 - 11x + 2) ?$$

3. Below you will find a collection of rectangles.

   (a) Define a mathematical measure that allows you to tell which rectangle is the "most square" and which is the "least square."

   (b) Define a different measure that achieves the same result.

   (c) Is one measure "mathematically superior" to the other? Argue why, and be prepared to defend your choice to the class.



4. Mr. Jones meets Mr. Smith at a conference. They're mathematicians, so their conversations are often a bit weird:

   J: How are your kids?
   S: All three are just fine.
   J: I forget; how old are they?
   S: The product of their ages is 36.
   J: That's not enough information. How about the sums of their ages?
   S: That wouldn't give you the answer.
   J: Well, tell me something else.
   S: Linda, the oldest, plays the piano
   J: Aha! They're ___, ___, and ___.

   How old are the kids? Ages are whole numbers, of course.

5. Two jars are on a table. One contains a liter of wine, the other a liter of water. Someone takes 3 tablespoons of wine and pours them in the water. The mixture is stirred, but not completely; the resulting solution is not homogeneous. Then someone else takes three tablespoons of the wine-in-water mixture, and pours them into the container of wine. Is there more wine in the water, or more water in the wine?

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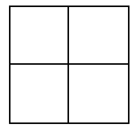
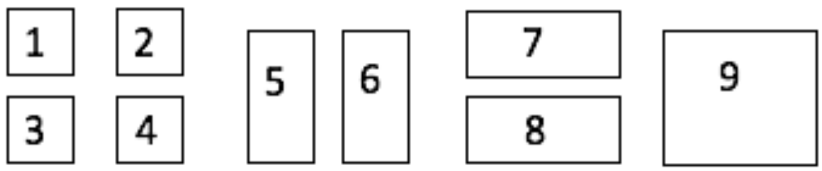
6. How many rectangles are there on an 8 x 8 chess board? Be careful to count them all -- any rectangle with its sides as grid lines in the chess board "counts." As an example, note that the 2x2 chess board,


yields



7. The game of NIM is played as follows. There are 25 objects in a pile. Player A goes first, removing 1 to 3 objects from the pile. Then player B goes, removing from 1 to 3 objects; then player A again, etc. The winner is the person who forces his (or her) opponent to take the last object. Does either A or B have a strategy that guarantees a win?

8. In an elimination tournament in tennis, the players in any round are paired, and play one game against each other. All the losers are out of the competition, and the winners go on to the next round. If there are an even number of players in a round, things are simple. For example, with 28 players in a round, there are 14 games; 14 people are removed from the tournament and 14 go on. If there are an odd number of players, one person does not play but does go on to the next round. So if there are 29 players, there are 14 games; there are 14 losers, and 15 go on. In general, if K players begin a round:

   If K is even, there are K/2 games; K/2 are eliminated and K/2 go on.

   If K is odd, there are (K-1)/2 games; (K-1)/2 are eliminated and [(K-1)/2+1] or [(K +1)/2] go on.

   Of course, the tournament continues until the final game, when the two finalists play for the championship. The question: If N players begin the tournament, how many games (not rounds – that's another question) must be played before a winner is determined?

9. Suppose you pick 21 points on the boundary of a circle. You then draw all of the lines segments that connect pairs of those points. If the points have been chosen so that no three of the segments intersect at the same point (that is, the circle is divided into the maximum possible number of regions), into how many regions is the circle divided?